Torridon Primary School

Calculation Policy

INTRODUCTION

Torridon Primary School's Calculation Policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics and it provides guidance on appropriate calculation methods and progression. The content is set out in progressive blocks under the following headings: addition, subtraction, multiplication and division. This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Children will use mental methods as their first port of call when appropriate, but for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

AIMS OF THE POLICY

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations, which they can use with confidence and understanding

The principle of the concrete-pictorial-abstract (CPA) approach is; for pupils to have a true understanding of a mathematical concept, they need to master all three phases. Reinforcement is achieved by going back and forth between these representations. For example, if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial. In short, these are not 'exclusive' representations.

Practical - CPA

- Develop children's fluency with basic number facts, fluency in mental calculation and in the use of written methods
- Develop children's understanding of the = symbol; teach inequality alongside teaching equality
- Don't count, calculate
- Look for pattern and make connections
- Use efficient methods
- Use empty box problems
- Expose mathematical structure and work systematically
- Explore resources and methods and move between the concrete and abstract
- Contextualise the mathematics
- Use questioning to develop mathematical thinking
- Expect children to use correct mathematical terminology and speak in full sentences
- Identify difficult points/misconceptions

HOW TO USE THIS POLICY

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use assessment for learning to identify suitable next steps in calculation for groups of children
- If, at any time, children are making significant errors, return to the previous stage in the calculation policy
- Always use suitable resources, models and images to support children's understanding of calculation and place value, as appropriate
- Encourage children to explore efficient strategies when solving problems

Mathematical language

Children deepen their understanding by explaining, creating problems, justifying and proving using mathematical language. This acts as a scaffold for their thinking, deepening their understanding further.

'The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.'
(2014 Maths Programme of Study)

Mathematical thinking

Children deepen their understanding by giving examples, by sorting or comparing, or by looking for patterns and rules in the representations they are exploring problems with.

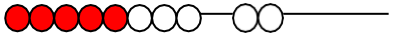
Multiple representations

Children deepen their understanding by representing concepts using objects and pictures, making connections between different representations and thinking about what different 5representations stress and ignore.

Reception - Addition

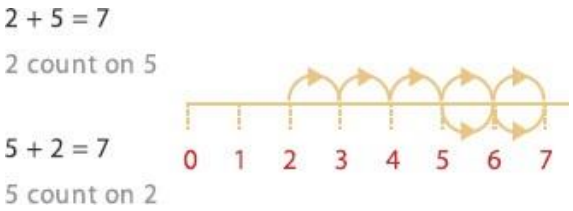
Concrete

Bead STRINGS or bead bars can be used to illustrate addition



8+2=10

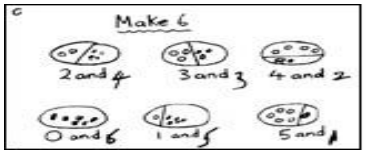
They use number lines and practical resources to SUPport calculation and teachers *demonstrate* the use of the number line.



Pictorial

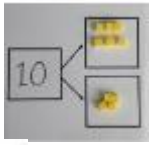
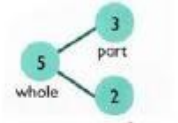
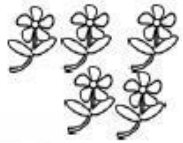
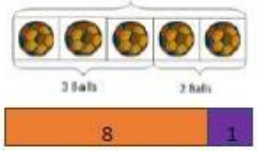
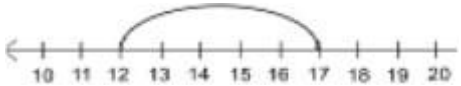
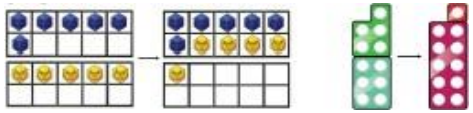
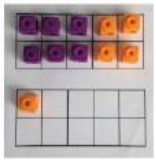
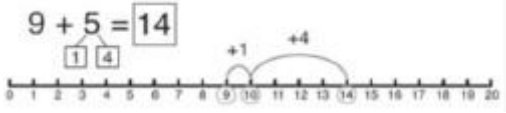
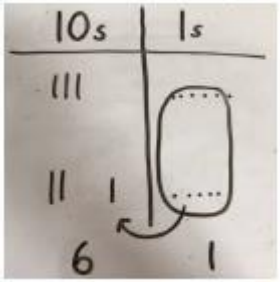
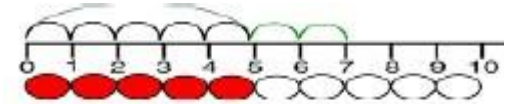
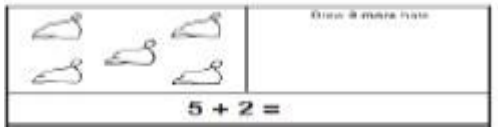
Children are encouraged to develop A mental picture of the number SYSTEM in their heads to use for calculation.

They develop ways of recording calculations using pictures, etc.

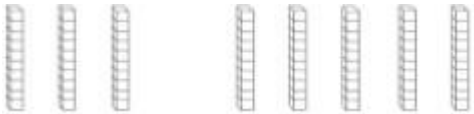
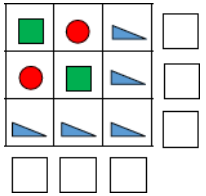
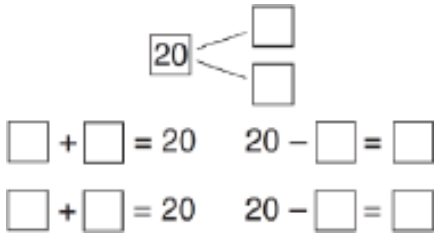
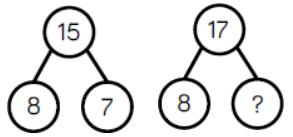
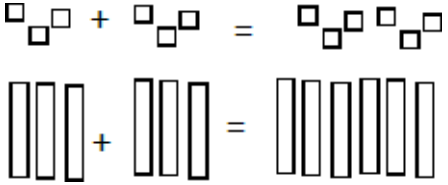
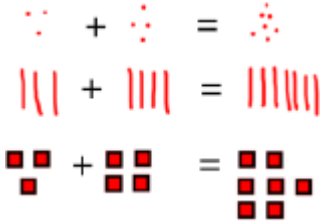
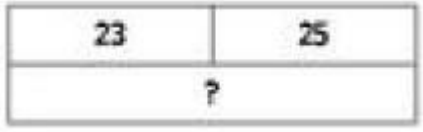
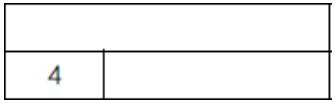
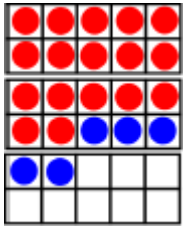
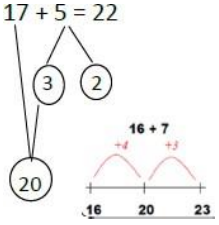
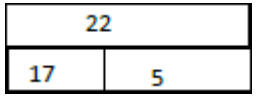
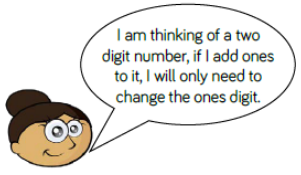


Expectation: All numbers are presented as SOON as possible on A line to SHOW the relationship between them

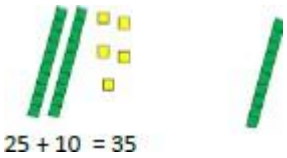
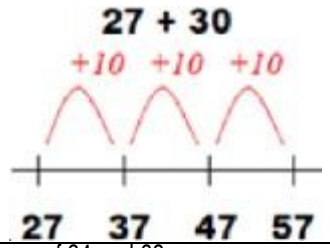
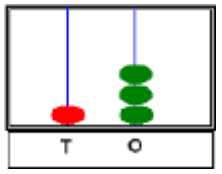
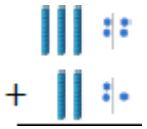
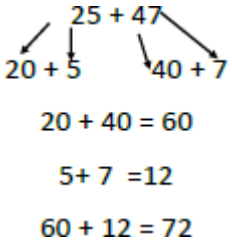
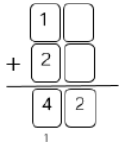
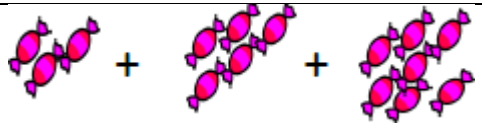
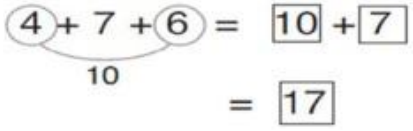
Year 1 - Addition

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Combining two parts to make A whole: part-whole model	  Use part-part-whole model. Use cubes to add two number together as A group or in A bar.  	   Use pictures to add two numbers together as A group or in A bar.	4 + 3 = 7  10= 6 + 4	There are 6 children.  How many different ways can you sort the children? Complete A part-whole model for each way. Can you partition the children into more than 2 groups?
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smallest number, 1 by 1, to find the answer. 	12 + 5 = 17  Start at the larger number on the number line and count on in ones, or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find the answer.	True or false? Explain why. "If I add 0 to A number, the number stays the same." Can you use A number line or counters to help you explain your answer?
Regrouping to make 10	Using ten frames and counters/cubes, or use Numicon. 6 + 5 =  Start with the bigger number and use the smaller number to make 10. 	 Use pictures or A number line. Regroup or partition the smaller number using the part-part-whole model to make 10. 	7 + 4 = 11 If I am at seven, how many more do I need to make to? How many more do I add on now?	Children to represent the base 10 in A place value chart. 
Represent and use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language: '1 more than 5 is equal to 6' '2 more than 5 is 7' '8 is 3 more than 5'	Kim has 3 number cards. 3 5 2 She has written two number sentences. 3 + 5 = 2 3 = 5 + 2 Explain what Kim has done wrong. Correct her number sentences and complete the fact families.

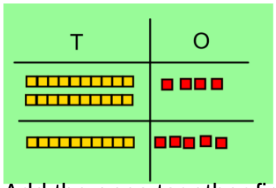
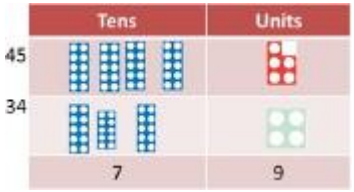
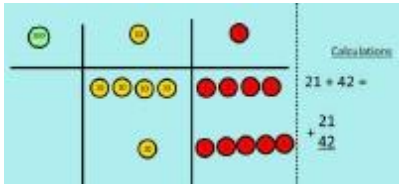
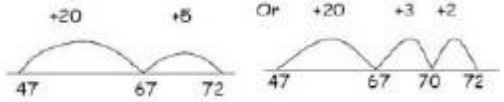
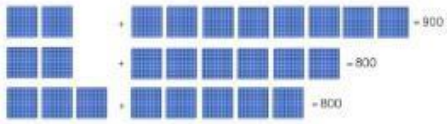
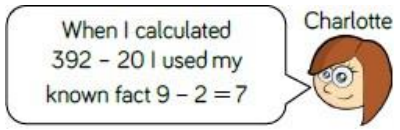
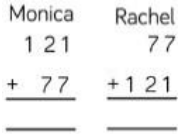
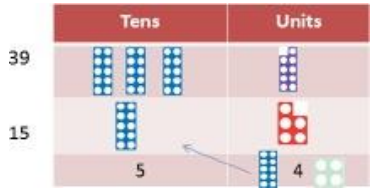
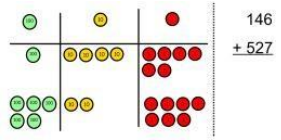
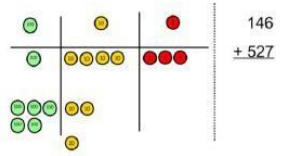
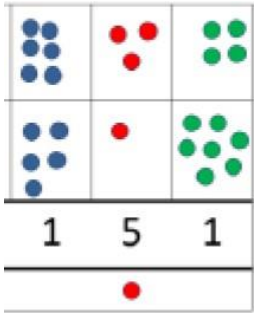
Year 2 - Addition

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Adding multiples of ten	50 = 30 + 20  Model using dienes and bead strings.	 3 tens + 5 tens = ___ tens 30 + 50 = ____ Use representations for base 10.	20 + 30 = 50 70 = 50 + 20 40 + ___ = 60	 Circles represent 20 Triangles represent 10 Squares represent 50 What is the value of each row and column?
Use known number facts Part-part-whole	 Children explore ways of making numbers within 20.	 20 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1	Deb thinks she knows the missing number without calculating the answer. Can you explain how this could be possible?  15 8 7 17 8 ?
Using known facts		 Children draw representations of H, T and O.	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 7000	Kim says, "If I know 9 + 1 = 10, I can work out 90 + ___ = 100". Find the missing number and explain how Kim knows.
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 25 ? 23 + 25 = 48	Here is an incomplete bar model. The total is greater than 10, but less than 20. What could the numbers be? How many different combinations can you find?  4
Add a two-digit number and ones	 17 + 5 = 22 Use ten frame to make 'magic ten'. Children explore the pattern: 17 + 5 = 22 27 + 5 = 32	 17 + 5 = 22 3 2 20 16 + 7 16 20 23 Use part-part-whole and number line to model.	17 + 5 = 22 Explore related facts: 17 + 5 = 22 5 + 17 = 22 22 − 17 = 5 22 − 5 = 17  22 17 5	Always, sometimes, never?  I am thinking of a two digit number, if I add ones to it, I will only need to change the ones digit. Explain your answer.

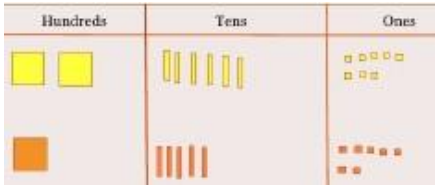
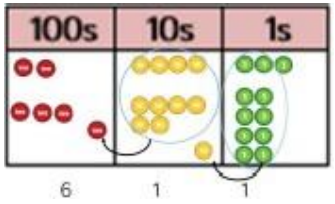
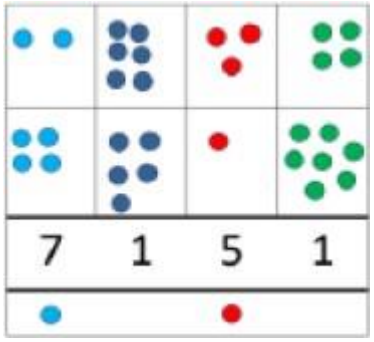
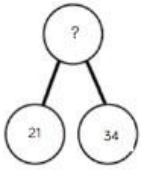
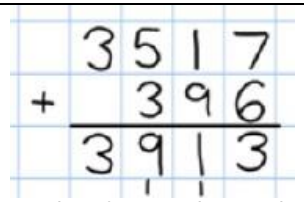
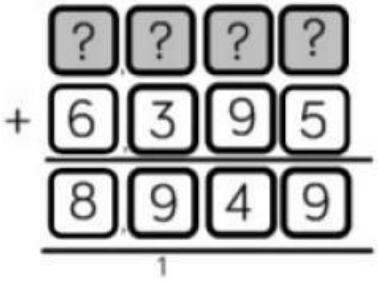
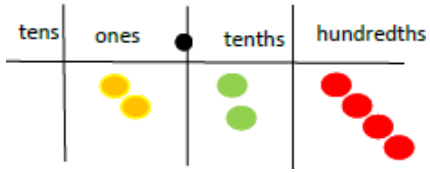
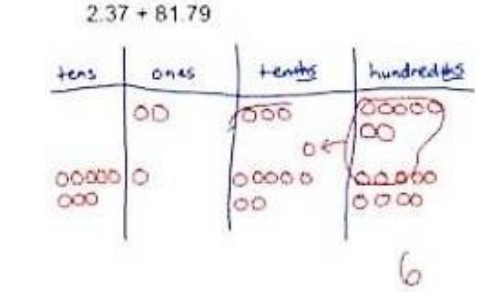
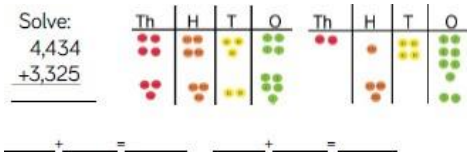
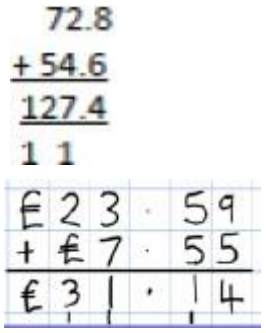
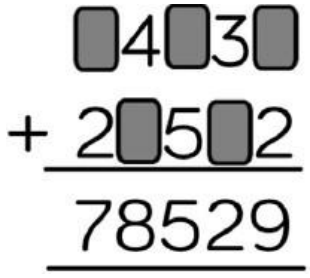
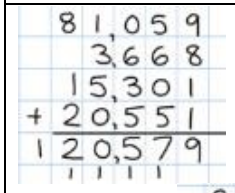
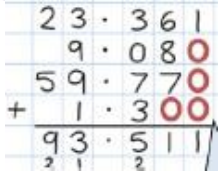
Year 2 - Addition

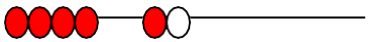
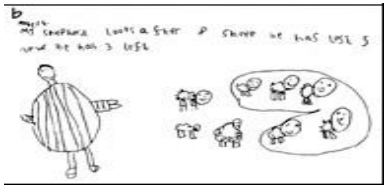
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Add A 2-digit number and tens	 Explore that the ones digit does not change.		27 + 10 = 37 27 + 20 = 47 27 + __ = 57	 Tom has three spare red beads. What numbers could he make? Explain your answer.
Add two 2-digit numbers	 Model using dienes, place value counters and Numicon.	Find the sum of 34 and 23 		How many different ways can you solve 19 + 11? Explain your method to A partner. Use concrete or pictorial resources to help explain your method.  Find all the possible pairs of numbers that can complete the addition. How do you know you have found all the pairs? What is the same about all the pairs of numbers?
Add three 1-digit numbers	 Combine to make 10 first if possible, or bridge 10 then add third digit.	 Regroup and draw representation. 	 Combine the two numbers that make/bridge ten, then add on the third.	Always, sometimes, never? odd + odd + odd = odd Use one-digit numbers to test if this is true. E.g. 3 + 5 + 7 Take 3 consecutive one-digit numbers, e.g. 4, 5 and 6. Add them together. What do you notice? Choose different groups of 3 consecutive one-digit numbers and see if there is A pattern.

Year 3 - Addition

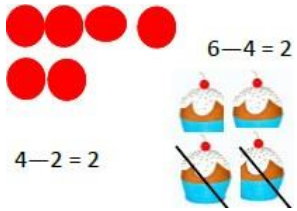
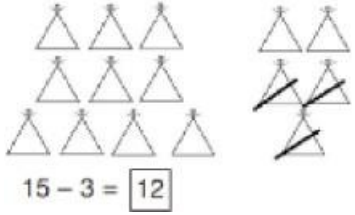
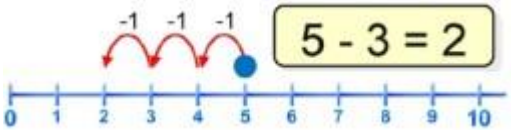
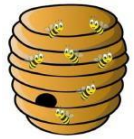
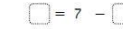
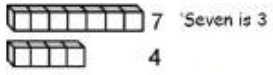
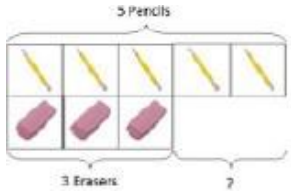
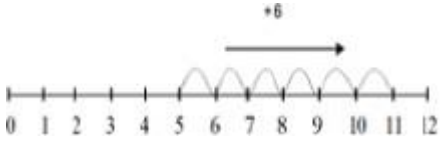
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Column addition – no regrouping ('friendly numbers') Add two or three 2/3-digit numbers	 Model using dienes or Numicon. Add the ones together first, then the tens.   Move to using place value counters.	Children move to drawing the counters using a tens and one frame.   Use a number line and bridge ten using part-whole if necessary.	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.	Which is the odd one out? Explain why.   Explain Charlotte's method. Monica and Rachel have some sweets. Monica has 77 and Rachel has 121. They have written the calculation differently. Who is correct? Why? 
Column addition – with regrouping	 Exchange ten ones for a ten. Model using Numicon and place value counters.  	 Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line.	20 + 5 40 + 8 60 + 13 = 73 Start by partitioning the numbers before the formal column method, to show the exchange and regrouping. 536 + 85 621 1 1	Khloe  Here is her working out. 2 6 5 + 8 7 2 4 2 Is she correct? Explain why. Joan and Fred play a game. Fred scored 354. Joan scores 478. What was the total of their scores? Car A drives 248 miles. Car B drives 40 miles more. How many miles do they drive altogether?

Years 4 - 6 – Addition

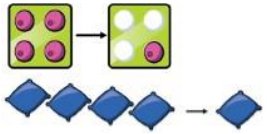
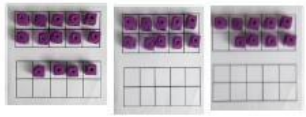
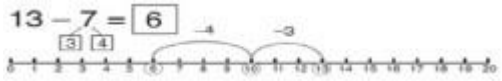
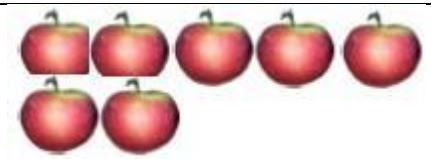
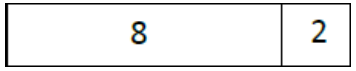
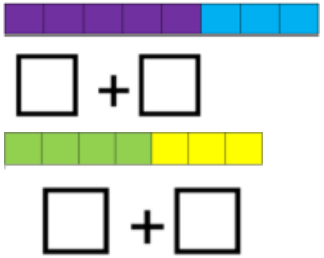
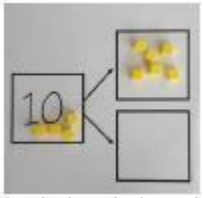
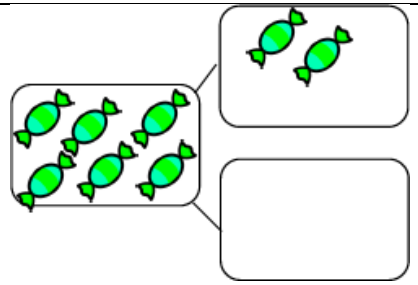
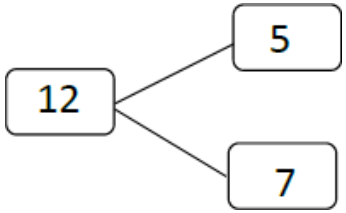
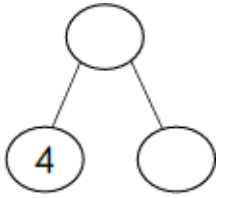
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Year 4 – add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for A ten, ten tens for A hundred and ten hundreds for A thousand.   Children can also use place value arrow cards.	Draw representation using place value grid.   ? | 21 | 34	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures.	What is the missing four-digit number? 
Year 5 – add numbers with more than 4 digits Add decimals with 2 decimal places, including money	As Year 4.  Introduce decimal place value counters and model exchange for addition. Cubes can be used to create bar models.	  	 	Work out the missing numbers. 
Year 6 – add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal places			 Insert zeros for place holders.  	Here is A bar model. A | B 631,255 A is an odd number which rounds to 100,000 to the nearest ten thousand. I t has A digit total of 30. B is an even number which rounds to 500,000 to the nearest hundred thousand. It has A digit total of 10. A and B are both multiples of 5 but end on different digits. What are the numbers?

Reception - Subtraction	
Concrete	Pictorial
Bead STRINGS or bead bars can be used to illustrate SUBtraction including bridging through ten by counting back 3 then counting back 2.  6 – 2 = 4 They use number lines and practical resources to SUPport calculation. Teachers *demonstrate* the use of the number line. Top tip: Recognise that there are at least 5 contextual interpretations of SUBtraction that need to be taught. Avoid over-emphasis on 'take-away'	Children are encouraged to develop A mental picture of the number SYSTEM in their heads to use for calculation. They develop ways of recording calculations using pictures etc. 

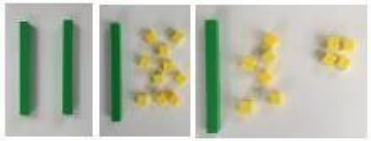
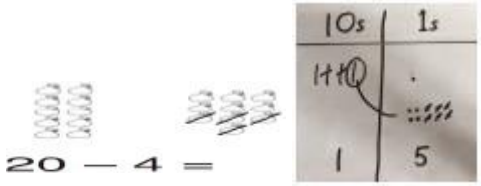
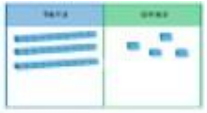
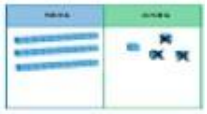
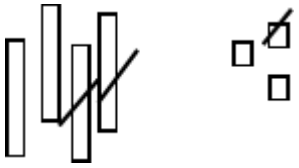
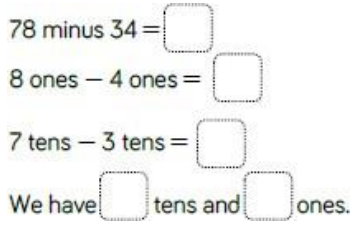
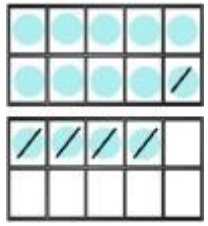
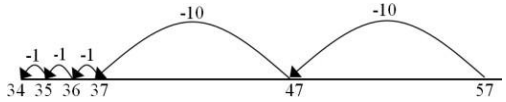
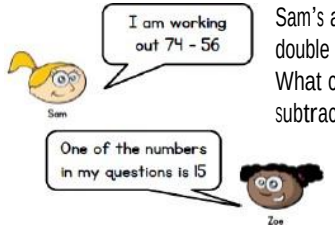
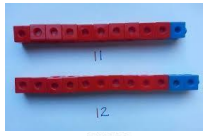
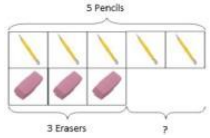
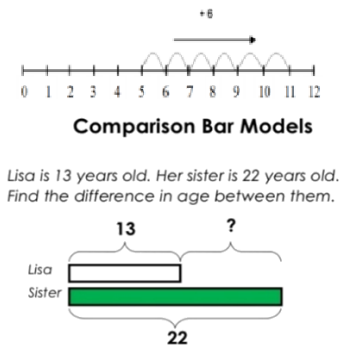
Year 1 – Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Taking away ones	Use physical objects, counters, cubes, etc. to SHOW how objects can be taken away. 	 Cross out drawn objects to SHOW what has been taken away.	7 – 4 = 3 16 – 9 = 7	There were 7 birds in a tree and 3 fly away.  At first there were___birds in the tree. Then___flew away. Now there are___birds in the tree. Some cakes have been eaten. There are 2 cakes left.  How many cakes could there have been and how many could have been eaten to be left with 2? Explain your reasons.
Counting back	 Move objects/counters away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using A number line.	Put 13 in your head, count back 4. What number are you at?	 How many calculations can you complete? Why can't the digits 8 or 9 be used?  Tami is calculating 7 – 2 and does this by counting backwards on a number line. She gets an answer of 6  What mistake has she made?
Find the difference	Compare objects and amounts.  'Seven is 3 more than four' 'I am 2 years older than my sister'  Lay objects to represent A bar model.	Count on using A number line to find the difference. 	Hannah has 12 SWEETS and her sister has 5. How many more does Hannah have than her sister?	Two numbers have A difference of 4. The larger number is less than 10. What could the two numbers be?

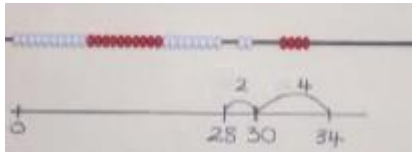
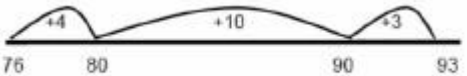
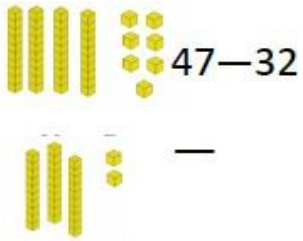
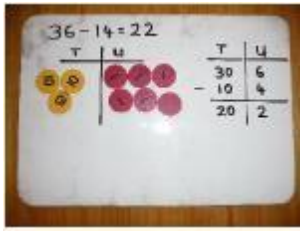
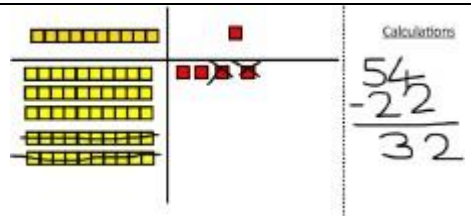
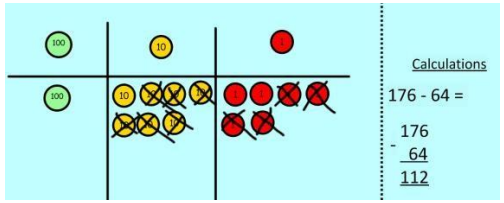
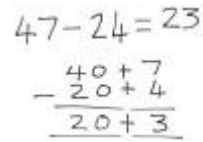
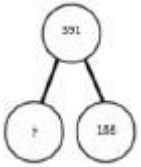
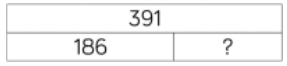
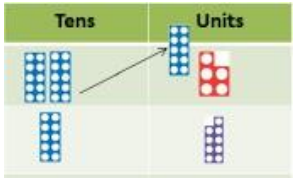
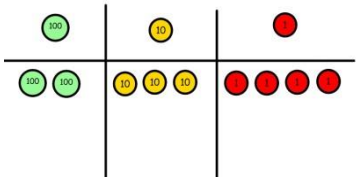
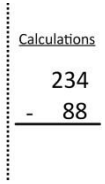
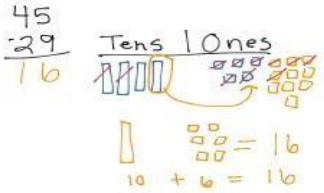
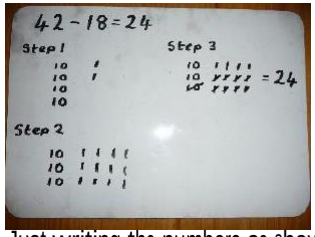
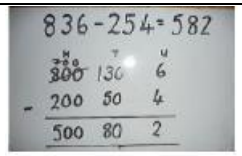
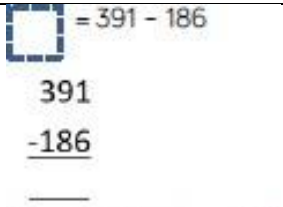
Year 1 – Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Make 10	4 – 3 = 1  14 – 9  Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	13 – 7  Jump back 3 first, then another 4. Use ten as the stopping point.	16 – 8 How many do we take off first, to get to 10? How many left to take off?	There are 14 ducks in a pond. 6 of them fly away. Create number sentences to represent this story. How many ducks are left in the pond?
Bar model	 5 – 2 = 3		 10 = 8 + 2 10 = 2 + 8 10 – 2 = 8 10 – 8 = 2	
Represent and use number bonds and related subtraction facts within 20 Part-part-whole model (PPW)	 Link to addition. Use PPW model to show the inverse. If 10 is the whole and 6 is one of the parts, what is the other part? 10 – 6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the PPW model. 	 A butterfly's spots have fallen off. How many different ways can you put the spots back on? Using the digits 0 – 9, how many PPW models can you complete?  One of the parts always has to be 4. You can only use each digit once. Explain why you can't use 0. What other digits can't you use and why?

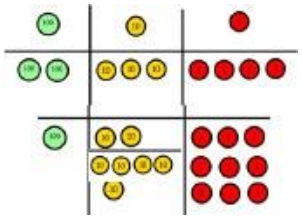
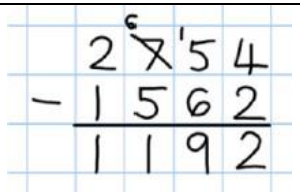
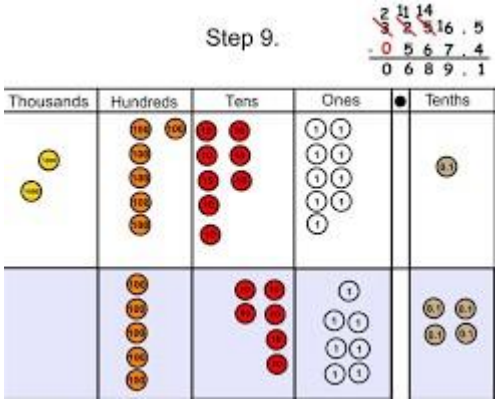
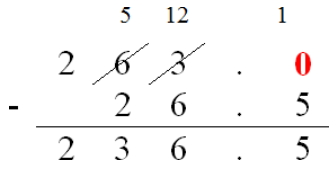
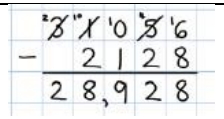
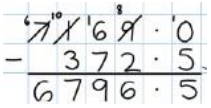
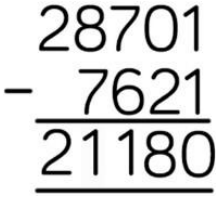
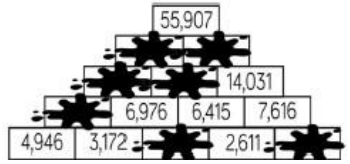
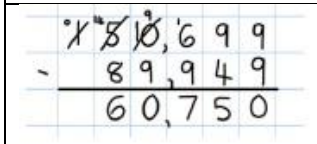
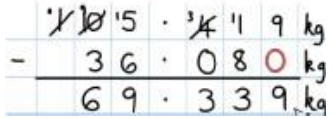
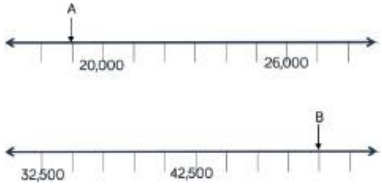
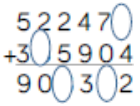
Year 2 – Subtraction

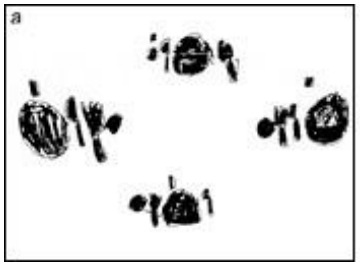
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Regroup a ten into ten ones	 Use PV chart to show how to change a ten into ten ones, use the term 'take and make'.		20 – 4 = 16	Jack has 22 sweets. He eats 7 sweets. How many sweets does he have left?  Partition 7 into 5 and 2, use this to bridge the 10.
Partitioning to subtract without regrouping 'Friendly numbers'	   34 – 13 = 21 Use dienes to show how to partition the number when subtracting without	Children draw representations of dienes and cross off.  43 – 21 = 22	43 – 21 =22	
Counting back/bridging	 Children to present the ten frame using counters and discuss.what they did to make/bridge through ten.		24 – 15 = 9 | | 4 11	Sam and Zoe are working out some subtractions.  Sam's answer is double Zoe's answer. What could Zoe's subtraction be?
Find the difference	Compare amounts and objects to find the difference.   Use cubes to build towers or make bars to find the difference. Use basic bar models with items to find the difference.	 Count on to find the difference. Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.	Jasmine has 33 stickers. Ollie has 54 stickers. How many more stickers does Ollie have? What method did you use to solve the problem?

Year 3 – Subtraction

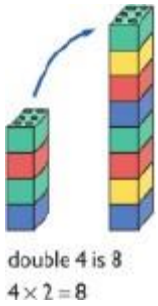
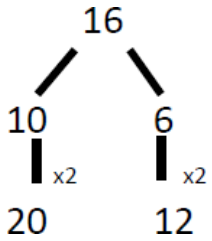
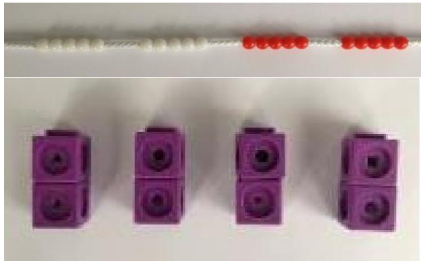
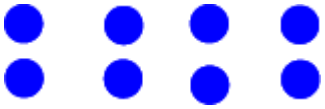
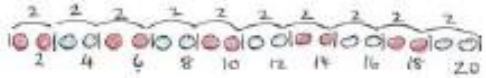
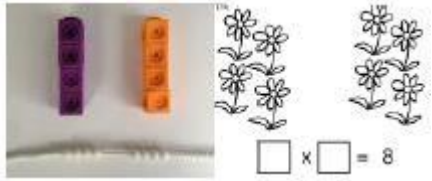
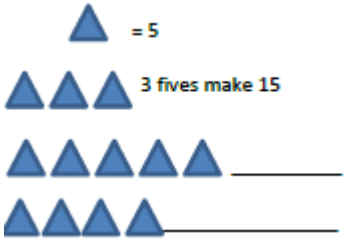
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Counting back/bridging	 Use a bead bar/string to model counting to next ten and then the rest.	 'counting on' to find 'difference' Use a number line to count on to the next ten and then the rest.	93 – 76 = 17 193 – 176 = 17	Raj spent £391, Timmy spent £186. How much more did Raj spend?
Column subtraction – without regrouping	 Use base 10 or Numicon to model.  Show how you partition numbers to subtract. Again make the larger number first.	 Draw representation to support understanding. 	  Intermediate step may be needed to lead to clear subtraction understanding, e.g. expanded column method.	 
Column subtraction – with regrouping	 Begin with base 10/Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make'. Make the larger number with the place value counters.  	 Children may draw base 10/PV counters and cross off.  When confident, children can find their own way to record the exchange/ regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Begin by partitioning into PV columns.  Then move to formal.	

Years 4 - 6 – Subtraction

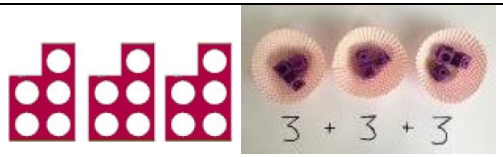
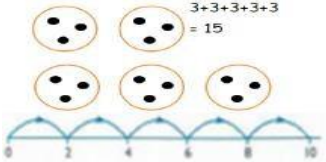
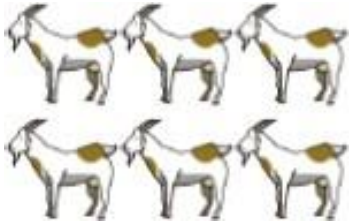
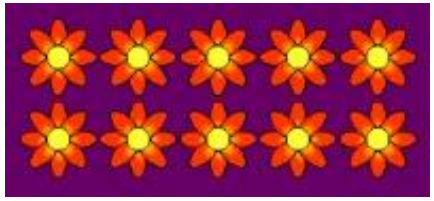
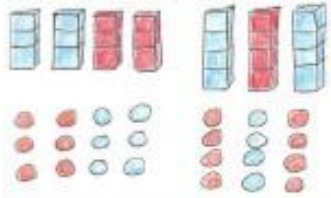
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Year 4 – Subtracting tens and ones Up to 4-digits Introduce decimal subtraction through the context of money	234 – 179  Model process of exchanging using Numicon, base 10 and then move to PV counters.	Children to draw PV counters and SHOW their exchange – see Year 3.		Three primary SCHOOLS join together to go on A school visit to The Deep in Hull. 1,235 people go on the trip. There are 1,179 children and 27 teachers. The rest are parents. How many parents are there? What do you need to do first? Which operation do you use?
Year 5 – Subtract with at least 4-digits, including money and measures Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal	Children use PV counters in A grid and carry out exchanges. 	Children to draw PV counters and SHOW their exchange – see Year 3 Moving forward the children use A more compact method. This will lead to an understanding of SUBtracting any number including decimals. 	 Use zeros for place holders 	 Holly has completed this SUBtraction incorrectly. Explain the mistake to Holly and correct it for her. Complete the pyramid using addition and subtraction. 
Year 6 – Subtract with increasingly large and more complex numbers and decimal values			 	Find the difference between A and B.  All the missing digits are the SAME. Find the missing digits. 

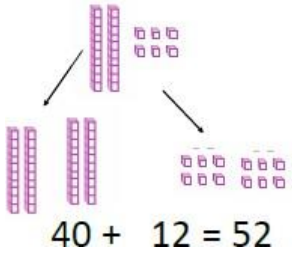
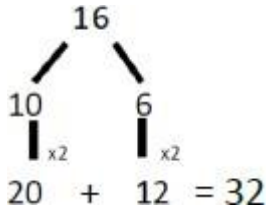
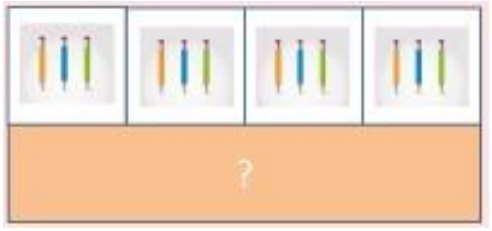
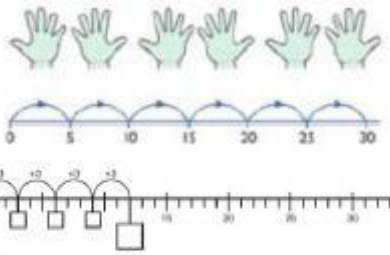
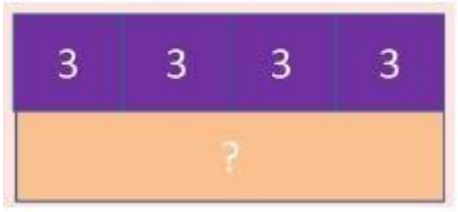
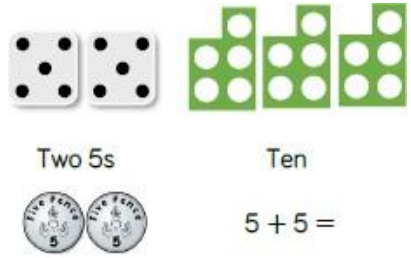
Reception - Multiplication	
Concrete	Pictorial
They will work on practical problem solving activities involving equal sets or groups. e.g. laying the table for the 3 bears and goldilocks  Begin to introduce children to the visual images of arrays – using real-life examples (brick work, paving slabs, windows in a building, anything with a repeating pattern in rows and columns!)	Children will experience **equal groups** of objects. They will count in 2s and 10s and begin to count in 5s. Children to draw representations of arrays.

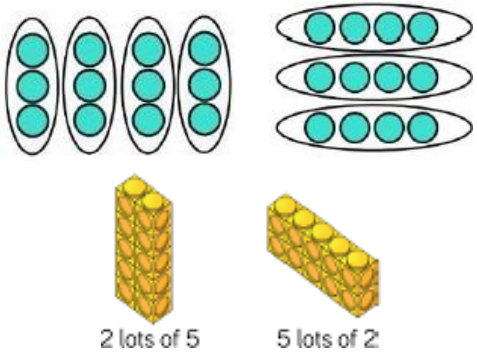
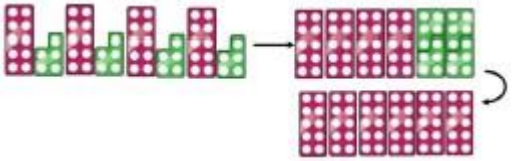
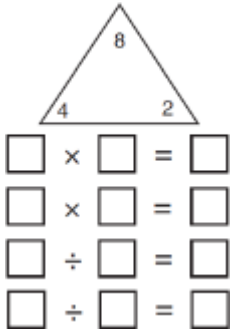
Year 1 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Doubling	 Use practical activities using manipulatives, including cubes and Numicon to demonstrate doubling.	Draw pictures to SHOW how to double numbers. Double 4 is 8 	Partition A number and then double each part before recombining it back together. 	SaskiA SAys, "You can double any number but you can only halve SOMe numbers." Can you prove this using counters or explain this to me?
Counting in multiples	Count the groups as children are skip counting. Children may use their fingers as they are skip counting. 	 Children make representations to show counting in multiples. 	Count in multiples of A number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30	Are there any numbers (multiples) in the 2 times table that are also in the 5 and 10 times table? Have you found them all? Have you used A STrategy to find them all?
Making equal groups and counting the total	Use manipulatives to create equal groups. 	Draw and make representations. Draw  to show 2 x 3 = 6	2 x 4 = 8 Questions displayed as word problems.	Fill in the sentences below. One has been done for you.  If the  = 2, what would your sentences be?

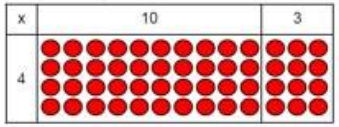
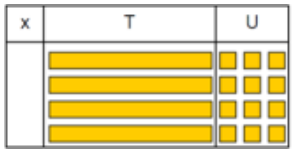
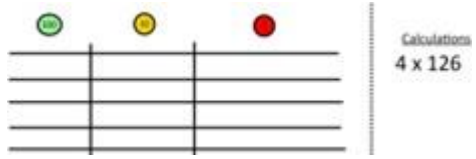
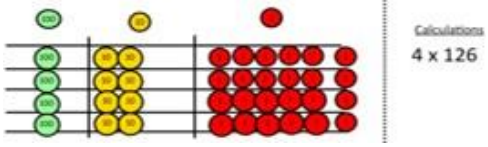
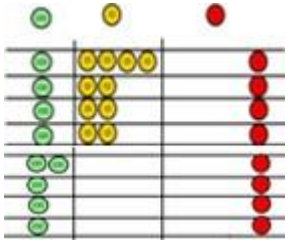
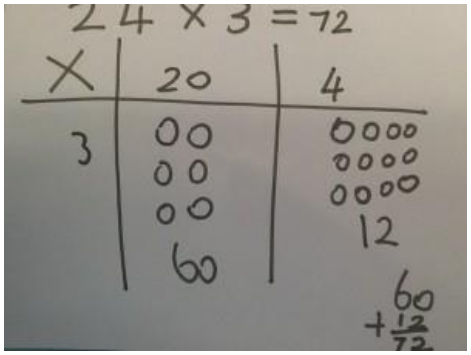
Year 1 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Repeated addition	 Use different objects to add equal groups.	Use pictorial including number lines to solve problems. There are 3 sweets in one bag. How many sweets are in 5 bags? 	Write addition sentences to describe objects and pictures. 	6 goats have twins. How many goats are born? 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2, etc. 	Draw representations of arrays to show understanding. 	3 x 2 = 6 2 x 5 = 10 Multiplications presented as word problems.	Here is an array.  Mandy says, "I can find four facts from this." Do you agree? Convince me.

Year 2 – Multiplication				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Doubling	Model doubling using dienes and PV counters.  	Draw pictures and representations to show how to double numbers.	Partition a number and then double each part before recombining it back together.  	True or false? 5 + 5 = 2 + 2 + 2 + 2 + 2 Draw an image or use cubes to help you explain your answer.
Counting in multiples of 2, 3, 4, 5, 10 from 0. (repeated addition)	Count the groups as children are skip counting. Children mau use their fingers as they are skip counting. Use bar models.    	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 4 x 3 = ___	Which one does not belong?  What do we need to change to make them all represent the same?

Year 2 – Multiplication				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Multiplication is commutative	Create arrays using counters, cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative.  	Use representations of arrays to show different calculations and explore commutativity.  2 lots of 5 5 lots of 2	12 = 3 x 4 12 = 4 x 3 Use and array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 =15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15	Find the product of 6 and 23 6 × 23 = □ = 6 × 23 6 × 23 23 × 6
Using the inverse *This should be taught alongside division, so that pupils learn how they work alongside each other*			2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.	Completing multiplication word problems and using the inverse to check and vice versa.

Year 3 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Grid method	Showing the links with arrays to first introduce the grid method.  4 rows of 10 4 rows of 3 Move on to base ten to move towards a more compact method.  4 rows of 13 Move on to PV counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones, making any exchanges needed to find the answer. 	Children can represent the work they have done with place value counters, in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different PV columns to show their thinking, as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. 	×	30	5
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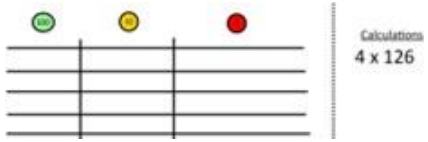
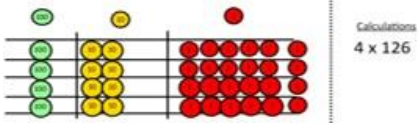
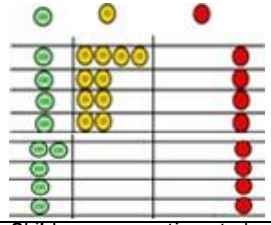
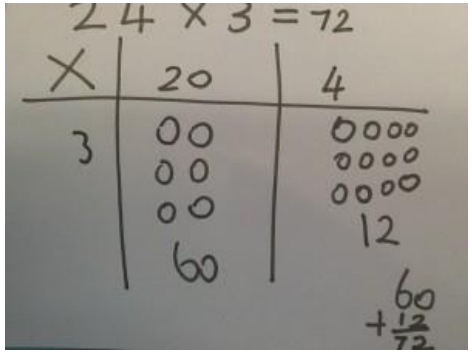
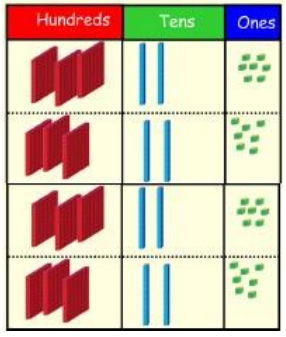
7	210	35 | **210 + 35 = 245** Moving forward, multiply by a 2 digit number showing the different rows within the grid method. |	10	8
---	---	---
10	100	80
3	30	24		Fill in the missing boxes. |	10	
---	---	---
5		40	 Explain your answer. Hassam is calculating 32 x 5. He writes his answer 15010. Can you work out Hassan's mistake and write an explanation of how he could do it correctly?

Year 4 – Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Grid method recap from Year 3 for 2-digits x 1-digit Move to multiplying 3-digit numbers by 1-digit. (Year 4)	Move on to PV counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Fill each row with 126.  Add up each column, starting with the ones, making any exchanges needed to find the answer. 	Children can represent the work they have done with place value counters, in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different PV columns to show their thinking, as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | ×	30	5
---	---	---
7	210	35	 210 + 35 = 245	Mrs While's class is selling tickets at £2 each for the school play. The class can sell one ticket for each chair in the hall. There are 7 rows of chairs in the hall. Each row contains 9 chairs. How much money will they make?
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This is initially done where there is no regrouping.  It is important at this stage that they always multiply the ones first and note down their answer, followed by the tens, which they note below.	The grid method may be used to show how this relates to formal written method. | x	300	20	7
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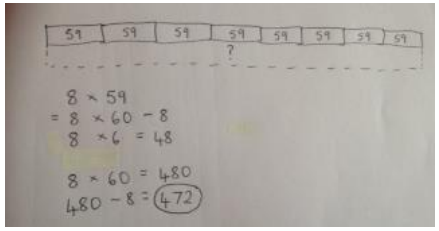
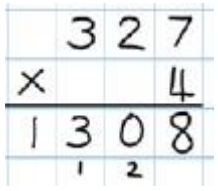
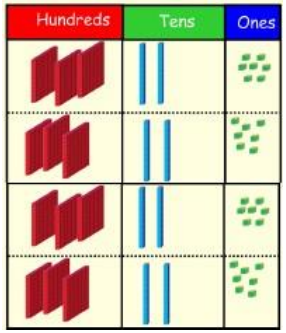
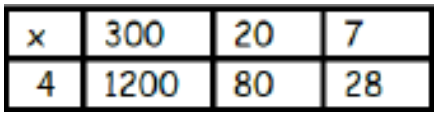
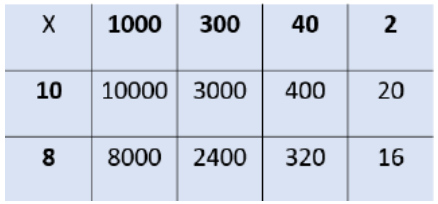
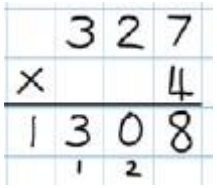
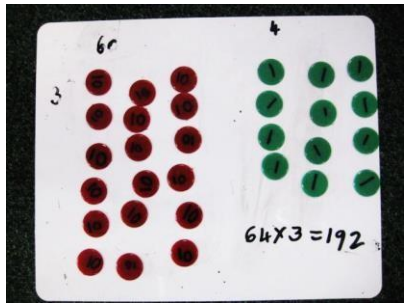
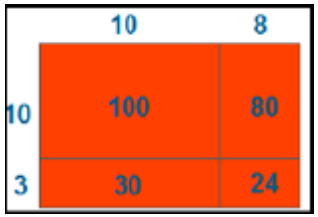
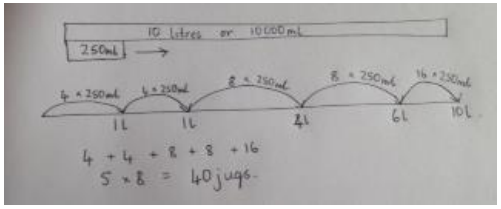
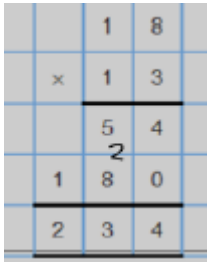
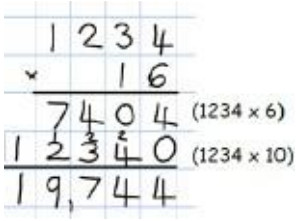
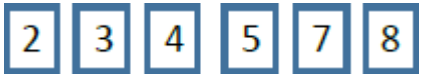
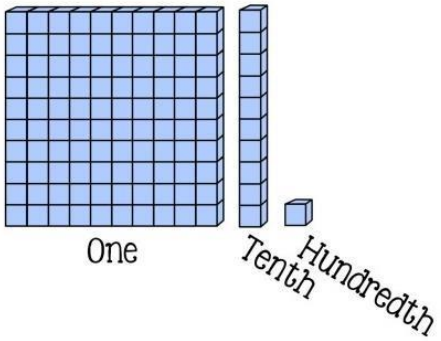
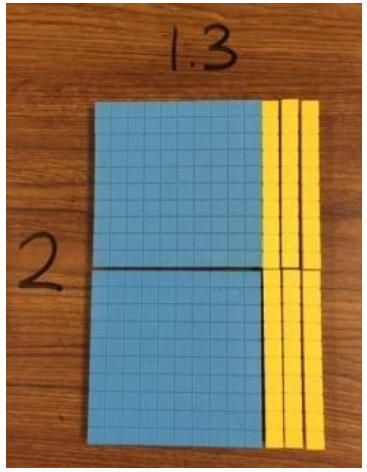
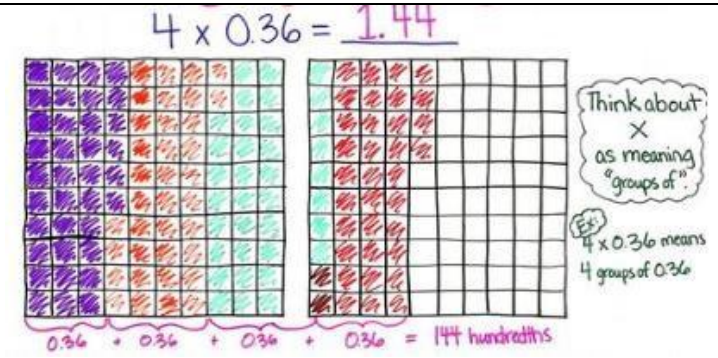
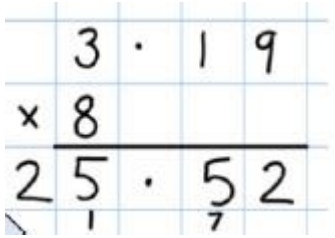
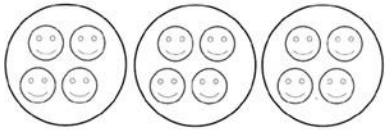
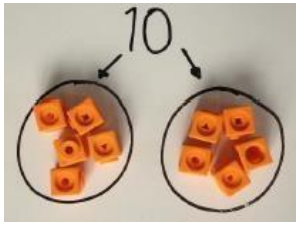
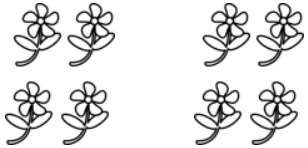
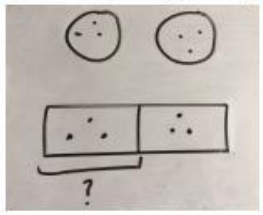
4	1200	80	28 | Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.  	32 x 24 8 (4 x 2) 120 (4 x 30) 40 (20 x 2) 600 (20 x 30) 768  327 x 4 28 80 1200 1308 This may lead to a compact method.  	Tom ate 9 grapes at the picnic. Sam ate 3 times as many grapes as Tom. How many grapes did they eat altogether? Sally has 9 times as many football cards as Sam. Together they have 150 cards. How many more cards does Sally have than Sam?

Years 5 & 6 – Multiplication

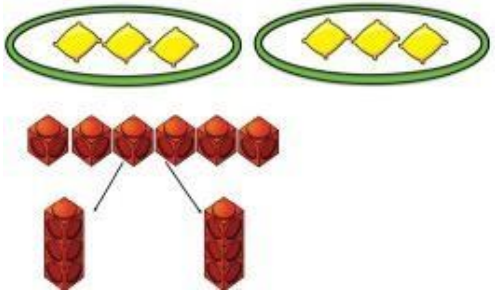
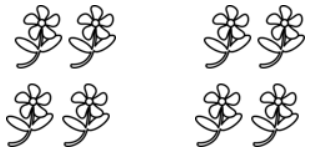
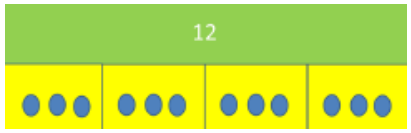
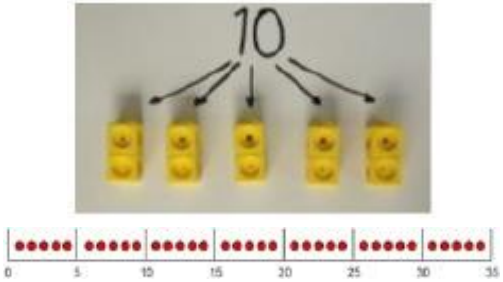
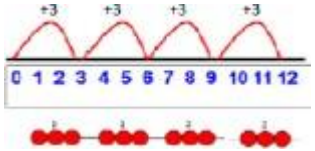
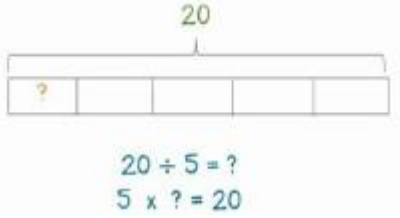
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Column multiplication for 3 and 4-digits x 1-digit (Multiply whole numbers and those involving decimals by 10, 100 and 1000)	It is important at this stage that they always multiply the ones first. Children can continue to be supported by PV counters. This is initially done where there is no regrouping. 	 	327 x 4 28 80 1200 1308 This will lead to a compact method. 	Emily has £1020 in her bank account and Philip has £120 in his bank account. Emily says, "I have ten times more money than you." Is Emily correct? Explain your reasoning.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside. 	 Continue to use bar modelling to support problem solving. 	18 x 3 on the first row. (8 x 3 = 24, 'carrying' the 2 for 20, then 1 x 3) 18 x 10 on the second row.  Show multiplying by 10, by putting zero in the ones column first. 	True or false? a) 5,463 x 18 is the same as 18 x 5,463 b) I can find the answer to 1,100 x 28 by using 1,100 x 30 and taking away two lots of 1,100 c) 70 ÷ 10 = 700 ÷ 100  Place the digits in the boxes to make the largest product. 

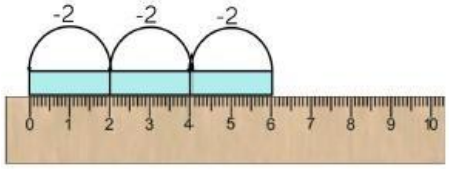
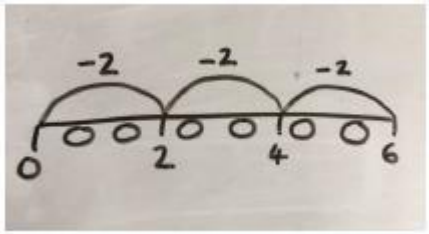
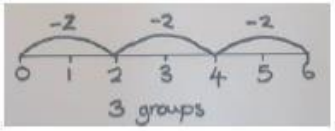
Year 6 – Multiplication				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Multiplying decimals up to 2 decimal places by a single digit.	Children can use dienes or base 10 to represent the numbers to be multiplied.  1.3 x 2 		Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and the answer. 	14 x 8 = 112 Use this to work out: 1.4 × 8 140 × 8

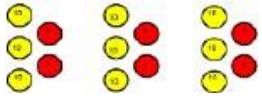
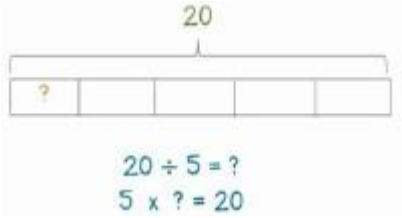
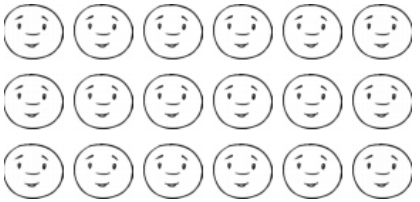
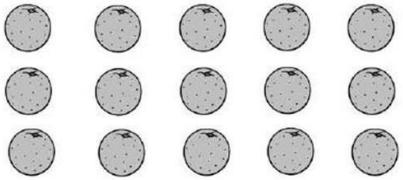
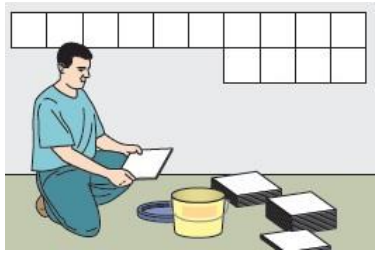
Reception - Division	
Concrete	Pictorial
Count in 2's to find out how many SOCKS are on the washing line:  Use A range of manipulatives to SHare into groups, SUCH as cubes, teddy bears, leaves, insects, etc.	Children will understand **equal groups** and SHare ITEMS out in play and problem solving. They will count in 2s and begin to count in 10s. 

Year 1 – Division				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Division as SHaring	   I have 10 cubes, can you SHARe them equally in to 2 groups?	Children use pictures or SHapes to SHARe quantities equally.  8 SHared between 2 is 4.  Represent the SHARing pictorially.	What is 12 SHared between 3? What is 10 SHared between 2?	6 ÷ 2 = 3 3 | 3 Children should also be encouraged to use their 2 times tables facts.

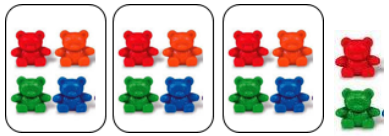
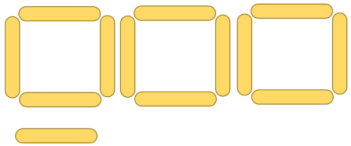
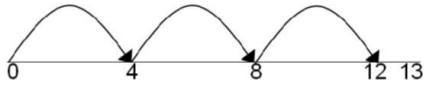
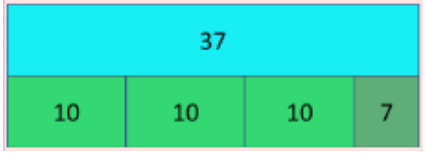
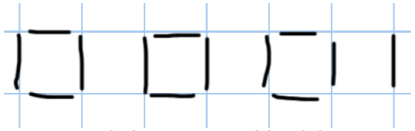
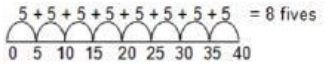
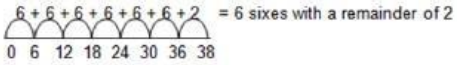
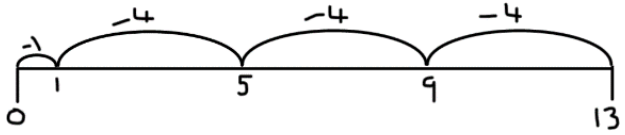
Year 2 – Division

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Division as sharing	Use a range of objects to share. 	Children use pictures or shapes to share quantities equally.  8 shared between 2 is 4. Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	Share 9 buns between three people. 9 ÷ 3 = 3	Two friends share 12 sweets equally between them. How many do they each get? Write this as a division number sentence.
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or PV counters to aid understanding. 	Use number lines for grouping.  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	Chocolate biscuits come in packs (groups) of 5. Sally wants to buy 20 biscuits in total. How many packs will she need to buy? Write this as a division sentence. Make up two more grouping stories like this one.

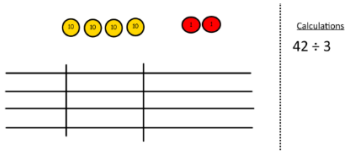
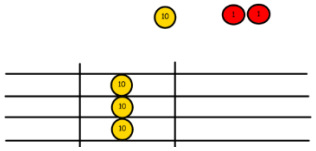
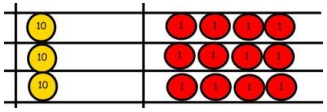
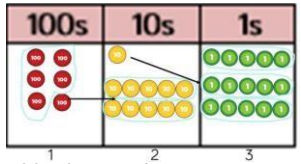
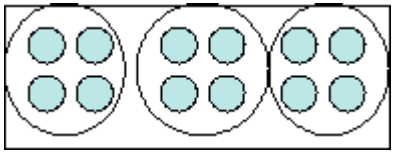
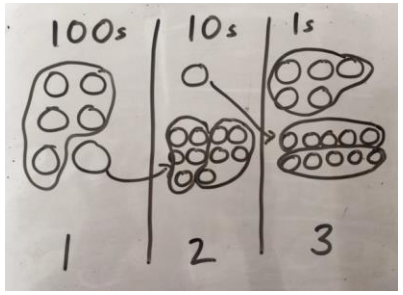
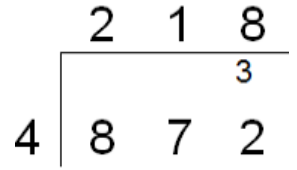
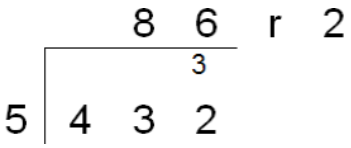
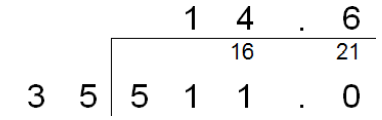
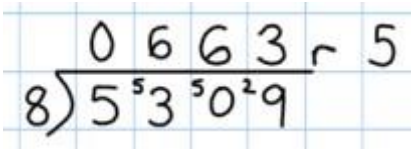
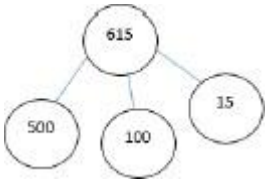
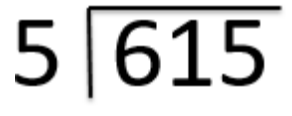
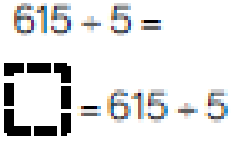
Year 2 – Division				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Repeated subtraction	Use Cuisenaire rods above a ruler. 6 ÷ 2  3 groups of 2	Children to represent repeated subtraction pictorially.  	Abstract number line to represent the equal groups that have been subtracted.  	I have 15 cookies. I want to put 3 cookies on each plate. How many plates do I need?

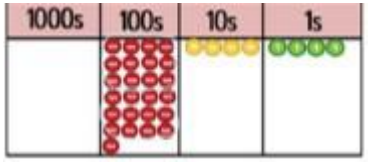
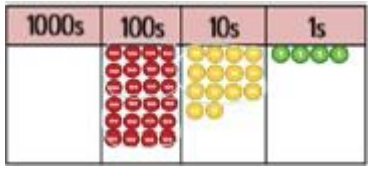
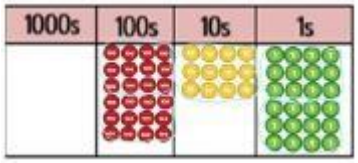
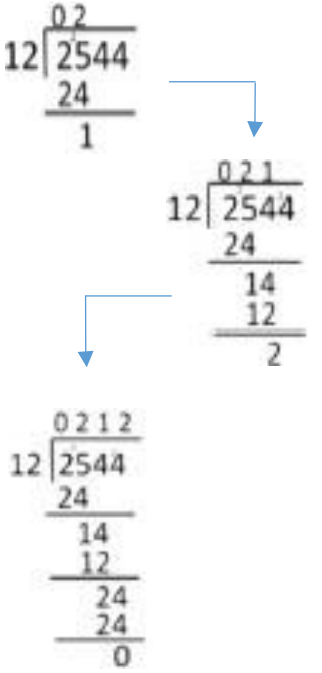
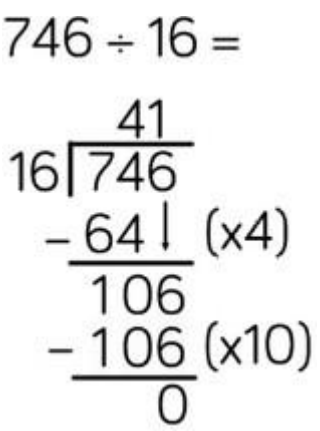
Year 3 – Division				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Division as grouping	Use cubes, counters, objects or PV counters to aid understanding.  24 divided into groups of 6 = 4 96 ÷ 3 = 32 	Continue to use bar modelling to aid solving division problems. 	How many group of 6 are in 24? 24 ÷ 6 = 4	How many division facts can you write for this group of peas? 
Division with arrays	 Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences. 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4	Roger is laying tiles. He has 84 tiles altogether. How many complete rows of tiles can he make? 

Year 3 – Division

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over.  Using lollipop sticks. Cuisenaire rods, above a ruler can also be used. 13 ÷ 4 Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.  There are 3 whole squares, with 1 left over.	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  Children to represent the lollipop sticks pictorially.  There are 3 whole squares, with 1 left over.	Complete written divisions and show the remainder using r.  Example without remainder: 40 ÷ 5 Ask "How many 5s in 40?"  Example with remainder: 38 ÷ 6  For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts. 13 ÷ 4 – 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over'. 	Share 10 cupcakes between 3 people. How many do they each get? Are there any left over?

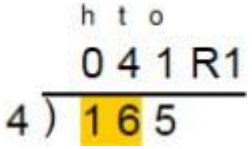
Years 4 - 6 – Division

Objective & Strategy	Concrete	Pictorial	Abstract	Application
Divide at least 3-digit numbers by 1-digit. Short division (Including remainders) (Years 5 & 6 Divide whole numbers and those involving decimals by 10, 100 and 1000.)	Use place value counters to divide using the bus stop method alongside.  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14. Using place value counters to group. 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently. Represent the PV counters pictorially.  	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Years 5&6 – Finally move into decimal places to divide the total accurately.   	Using the part-whole model, how can you divide 615 by 5, without using short division?  I have £615 and share it equally between 5 bank accounts. How much will be in each account? 615 pupils need to be put into 5 groups. How many will be in each group?  

Year 6 – Division				
Objective & Strategy	Concrete	Pictorial	Abstract	Application
Long division	Using PV counters. 2544 ÷ 12  We can't group 2 thousands into groups of 12, so we will exchange them.  We can group 24 hundreds into groups of 12, which leaves us with 1 hundred.  After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.  After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.	Children can draw PV grids and related representations.	 	Explain the mistake: 

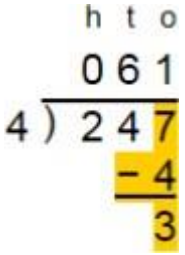
Year 6 – Long Division

Step 1 – A remainder in the ones.

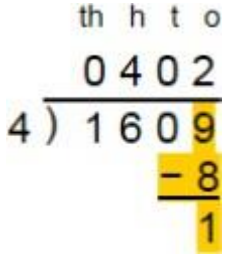


4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).
4 goes into 16 four times.
4 goes into 5 once, leaving A remainder of 1.

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).
8 goes into 32 four times (3,200 ÷ 8 = 400).
8 goes into 0, zero times (tens).
8 goes into 7 zero times and leaves A remainder of 7.



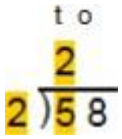
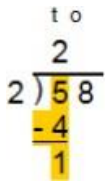
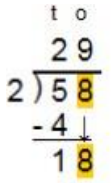
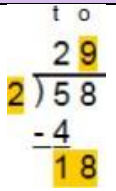
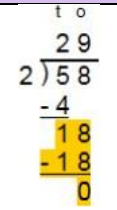
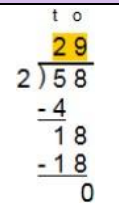
When dividing the ones, 4 goes into 7 one time.
Multiply 1 x 4 = 4, write that four under the 7, and SUBTract. This gives us the remainder of 3.
Check 4 x 61 + 3 = 247



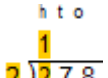
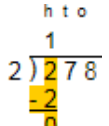
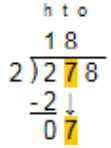
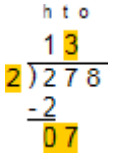
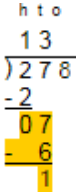
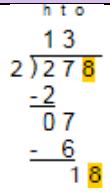
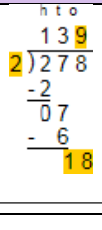
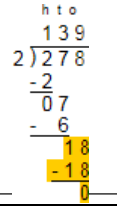
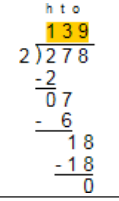
When dividing the ones, 4 goes into 9 two times
Multiply 2 x 4 = 8, write that eight under the 9 and SUBtract. This gives us the remainder of 1.
Check 4 x 402 + 1 = 1,609

Year 6 – Long Division

Step 2 – A remainder in the tens.

1. Divide		2. Multiply & SUBTRACt		3. Drop down the nextdigit	
	Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens, but there is A remainder.		Multiply 2 x 2 = 4, write that 4 under the five and SUBtract to find the remainder of 1 ten.		Drop down the 8 of the ones next to the leftover 1 ten. Combine the remainder ten with 8 ones, and get 18.
1. Divide		2. Multiply & SUBTRACt		3. Drop down the nextdigit	
	Divide 2 into 18. Place 9 into the quotient.		Multiply 9 x 2 = 18, write that 18 under the 18, and SUBTract.		The division is over SINCe there are no more digits in the dividend. The quotient is 29.

Step 2 – A remainder in any of the place values.

1. Divide		2. Multiply & SUBTRACt		3. Drop down the nextdigit	
	Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.		Multiply 1 x 2 = 2, write that 2 under the two, and SUBtract to find the remainder of zero.		Next, drop down the 7 of the tens next to the zero.
Divide		Multiply & SUBTRACt		Drop down the next digit	
	Divide 2 into 7. Place 3 into the quotient.		Multiply 3 x 2 = 6, write that 6 under the 7, and SUBTract to find the remainder of 1 ten.		Next, drop down the 8 of the ones next to the 1 leftover ten.
Divide		Multiply & SUBTRACt		Drop down the next digit	
	Divide 2 into 18. Place 9 into the quotient.		Multiply 9 x 2 = 18, write that 18 under the 18, and SUBTract to find the remainder of zero.		There are no more digits to drop down. The quotient is 139.

EYFS addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add More And Make Sum Total Altogether Score Double One more, two more, ten more How many more to make ….? How many more is … than …?	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base tens and ones	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 1 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Total More Tens Ones Digit	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base tens and ones	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 2 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Sum More than Total Altogether Plus Digit Partition into tens and ones	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html

	Base hundreds tens and ones Arrow Cards	
Year 3 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Sum More than Total Altogether Plus Partition into tens and ones Empty number line Count on Carry ten	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base hundreds tens and ones Arrow Cards	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 4 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Sum More than Total Altogether Plus Partition into tens and ones Empty number line Count on Carry ten Carry 100 Two digit three digit Crossing tens boundary Inverse	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base hundreds tens and ones Arrow Cards	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 5 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Sum More than Total Altogether Plus	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/

Partition into tens and ones Empty number line Count on Carry ten Carry 100 Two digit three digit Crossing tens boundary Inverse	Numicon Place Value Disks Cuisenaire Base hundreds tens and ones Arrow Cards	http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 6 addition		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Add Sum More than Total Altogether Plus Partition into tens and ones Empty number line Count on Carry ten Carry 100 Two digit three digit Crossing tens boundary Inverse	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base hundreds tens and ones Arrow Cards	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
EYFS subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Take (away) Leave How many are left/left over? How many have gone? One less, two less … ten less… How many fewer is … than Difference between Is the same as	100 square Number lines Number tracks Bead strings (for children) Bead bar Tens Frame Numicon Place Value Disks Cuisenaire Base tens and ones	https://www.coolmath4kids.com/manipulatives/base-ten-blocks https://mathsbot.com/manipulatives/placeValueCounters https://mathsbot.com/manipulatives/bar https://classroomsecrets.co.uk/free-year-1-part-whole-model-iwb-addition-and-subtraction-activity/ https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Ten-Frame/ http://www.ictgames.com/mobilePage/tenFrame/index.html
Year 1 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives

Year 2 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 3 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 4 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 5 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 6 subtraction		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
	A	
EYFS multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 1 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 2 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 3 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 4 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 5 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives
Year 6 multiplication		
Vocabulary	Models, Images and resources	Useful IWB links for manipulatives